SEMESTER-2 (Period-V)

TOPIC

5

Waves

P11CH5

5.1. INTRODUCTION

Energy can be transferred from one place to another through the bulk motion of matter. A running stream of water carries energy with itself as it moves along. There is another way of transferring energy in which there is no bulk motion of matter. This is by means of 'waves'. The waves are of three types—mechanical waves, electromagnetic waves and matter waves.

(*i*) **Mechanical waves** can be produced and propagated only in those material media which possess elasticity and inertia. These waves are also called elastic waves. Common examples include water waves, sound waves, and seismic waves. They can exist only within a material medium such as water, air and rock.

(*ii*) **Electromagnetic waves** do not require any material medium for their production or propagation. Common examples include visible and ultraviolet light, radio and television waves, microwaves, X-rays and radio waves. All electromagnetic waves travel through vacuum at the same speed c given by

$$c = 299792458 \text{ m s}^{-1} \approx 3 \times 10^{8} \text{ m s}^{-1}$$

(*iii*) **Matter waves** are waves associated with electrons, protons and other fundamental particles, and even atoms and molecules.

In this unit, we shall study only mechanical wave motion which will be referred to simply as wave motion.

5.2. WAVES ON SURFACE OF WATER

We can see and appreciate waves on a sea-shore. Waves can be generated in a large basin or a tub of water by just dropping a small stone or a pebble at the centre.

The dropped pebble creates a disturbance in the centre. The particles of water acquire energy (both kinetic and potential). This energy is transmitted to the next portion of the surface layer and so on. Thus we see something travelling outwards away from the source of disturbance in ever-expanding concentric circles (Fig. 5.1). In some regions, water level is below the usual normal level. These are called *troughs.* On either side of a trough, there are regions where water is at a level higher than the normal. These are called *crests.*

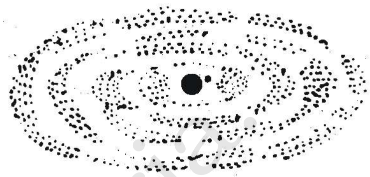


Fig. 5.1. Water waves

To sum up, *the disturbance moves progressively onwards in the form of alternate troughs and crests as shown in Fig. 5.2.* This disturbance is called 'wave'.

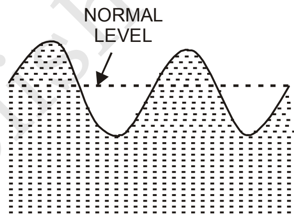


Fig. 5.2. Crests and troughs

It is the disturbance which travels outwards and not water. This fact can be verified by placing a piece of cork or a straw on the disturbed surface of water. It will be observed that the cork or straw just keeps on oscillating up and down about its mean position, sometimes riding a crest and at another time resting on a trough. The straw or cork will not move outwards with the disturbance.

Thus, the particles of the medium certainly oscillate about their mean positions but their permanent physical movement away from their original positions is not there.

***Wave motion** may be defined as a form of disturbance which is due to the repeated periodic vibrations of the particles of the medium about their mean positions and the motion is handed over from one particle to the other without any net transport of the medium.*

It may also be defined as under:

***Wave motion** is a means of transferring momentum and energy from one point to another without any transport of matter between the two points.*

5.3. WAVES IN STRINGS

Consider a stretched string tied at one end to a fixed support. Let the free end of the stretched string be given an upward jerk. This will produce an upward kink in the string. This upward kink travels, along the string, towards the fixed end as shown in Fig. 5.3.

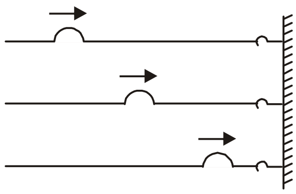


Fig. 5.3. Waves in strings

It may be noted that it is only the disturbance given to the free end that travels along the string and not any part of the string itself.

If the free end of the string is given one complete oscillation, then an upward kink will be followed by a downward kink along the string. However, if we continuously move the free end of the string up and down, a wave-train is observed to move, along the string, having alternate crests and troughs (Fig. 5.4).

Fig. 5.4. Wave-train in string

5.4. CHARACTERISTICS OF WAVE MOTION

(*i*) Wave motion is merely *a form of disturbance which is produced in the medium by the repeated periodic motion of the particles of the medium about their mean positions.*

(*ii*) The energy moves outwards away from the source while the particles of the medium continue vibrating about their mean positions with fixed frequency. Thus, a wave represents the transfer of energy from particle to particle. Energy can be transmitted over long distances by wave motion.

(*iii*) In order to set up wave motion in a medium, it is necessary that the medium should possess elasticity and inertia. Due to elasticity, the medium has a tendency to come back to its original condition. Due to inertia, the medium can store energy. The speed of a wave in a medium is determined by the inertia and elasticity of the medium. So, material media (having elasticity and inertia) are capable of transmitting mechanical waves. On the other hand, no material medium is necessary for the propagation of electromagnetic waves.

(*iv*) During their to and fro vibration about their mean positions, the particles possess different velocities. At the extreme position, the particle velocity is zero. The velocity increases as the particle moves towards the mean position. At the mean position, the particle velocity is maximum. The 'maximum velocity' is determined by the energy of the wave. On the other hand, a wave propagates with constant velocity in a homogeneous and isotropic medium.

To sum up, the wave velocity is very much different from the particle velocity.

(*v*) Depending upon the type of wave, the particles of the medium may actually oscillate up and down or the particles may move towards or against the direction of propagation of wave.

(*vi*) In a wave motion, all the particles of the medium do not start moving at once. But there is a constant phase difference between one particle and the next. The wave advances in that direction in which it meets particles with continuously decreasing phase. In simple words, the movement of each particle begins a little later than that of its predecessor.

5.5. TYPES OF MECHANICAL WAVES

Mechanical waves can be divided into two types :

(*i*) Transverse waves (*ii*) Longitudinal waves.

5.5.1. Transverse Wave Motion

Transverse wave motion *is that wave motion in which the individual particles of the medium execute simple harmonic motion about their mean positions in a direction perpendicular to the direction of propagation of the wave.* The wave itself is known as transverse wave.

The water waves, the movement of a kink in a rubber string, the movement of string in a '*sitar*' or a violin, the movement of the membrane of a 'tabla' or 'dholak' are all **examples** of transverse vibrations of these media and **transverse waves** generated in those media.

A transverse wave progresses as a series of troughs and crests. ***Crest*** *is the position of maximum displacement in the positive direction i.e., above the line of mean position or normal level.* As an example, in Fig. 5.5, A, C and E are crests. *When the displacement of a particle is maximum above the line of mean position, the particle is said to be at the crest of a wave.*

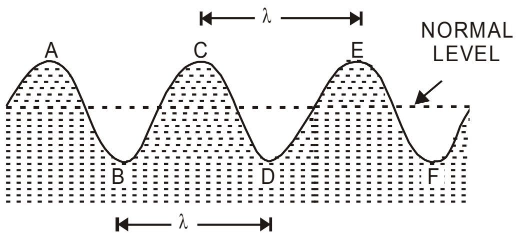


Fig. 5.5. Transverse wave

Trough *is the position of maximum displacement in the negative direction, i.e., below the line of mean position or normal level.* In Fig. 5.5, B, D and F are troughs. *When the displacement of a particle is maximum below the line of mean position, the particle is said to be at the trough of a wave.*

The distance between two consecutive crests or troughs is known as the wavelength.

Transverse waves can be transmitted through solids. They can also be set up on the surfaces of liquids. These waves cannot be transmitted inside liquids and gases. This is due to the fact that liquids and gases do not possess internal transverse restoring forces.

5.5.2. Longitudinal Wave Motion

Longitudinal wave motion *is that wave motion in which the individual particles of the medium execute simple harmonic motion about their mean positions along the direction of propagation of the wave.*

Sound wave is an example of longitudinal wave.

When a longitudinal wave travels through a medium, it produces compressions and rarefactions of the medium.

In a compression, the distance between any two consecutive particles of the medium is less than the normal distance.

So, the density of the medium in compression is more than the normal density.

In a rarefaction, the distance between any two consecutive particles of the medium is more than the normal distance. So, the density of the medium in a rarefaction is less than the normal density.

In Fig. 5.6 (*i*), the positions of different layers of air are shown when the tuning fork is not vibrating. However, when the tuning fork is set into vibration, the vibrating tuning fork sends out alternate waves of compression (or condensation) and rarefaction as depicted in Fig. 5.6 (*ii*). When these waves strike the ear drum of the listener, they make the ear drum

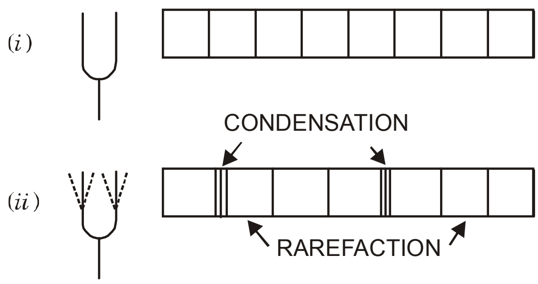


Fig. 5.6. Positions of different layers of air when (*i*) tuning fork is not vibrating (*ii*) tuning fork is vibrating

vibrate with the frequency of the incident waves. The distance between the centres of two nearest condensations or rarefactions is known as wavelength λ.

Again, consider the case of a spiral spring. When it is compressed at one end and released, the coils of the spring vibrate about their original positions along the length of the spring (Fig. 5.7). It will be observed that coils get closer together and move farther apart alternately. (AB + BC), *i.e.,* a compression and an adjoining rarefaction constitute one wave. Similarly, (BC + CD) or (CD + DE) or (DE + EF) constitute one wave.

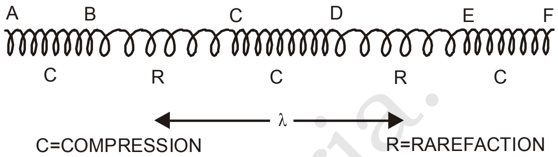


Fig. 5.7. Formation of compressions and rarefactions in spring

The succession of waves constitutes a wave train ABCDEF.

The longitudinal wave can be transmitted through solids, liquids or gases. In-fact, longitudinal wave is the only type of wave which can be propagated by a gas.

5.6. DISTINCTION BETWEEN LONGITUDINAL AND TRANSVERSE WAVES

Longitudinal Waves	Transverse Waves
1. The particles of the medium vibrate along the direction of propagation of the wave.	1. The particles of the medium vibrate at right angles to the direction of propagation of the wave.
2. The longitudinal waves travel in the form of alternate compressions (condensations) and rarefactions. One compression and one rarefaction constitute one wave.	2. The transverse waves travel in the form of alternate crests and troughs. One crest and one trough constitute one wave.
3. These waves can be formed in anymedium (solid, liquid or gas).	3. These waves can be formed in solids and on the surfaces of liquids only.
4. When longitudinal waves propagate, there are pressure changes in the medium.	4. When transverse waves propagate, there are no pressure changes in the medium.

5.7. IMPORTANT TERMS USED IN THE STUDY OF WAVE MOTION

(*i*) Crest. The elevation or hump caused in a medium due to the propagation of transverse wave through it is called crest.

(*ii*) Trough. The depression or hollow caused in a medium due to the propagation of transverse wave through it is called trough.

(*iii*) Compression. A portion of the medium where an increase in density occurs (because of reduction in volume) due to passage of longitudinal wave in it is called compression or condensation.

(*iv*) Rarefaction. A portion of the medium where a decrease in density occurs (because of increase in volume) due to passage of longitudinal wave in it is called rarefaction.

(*v*) Wavelength (λ). Following are the different ways of defining wavelength:

Wavelength of a wave is the distance travelled by the wave in a medium during the time a particle of the medium completes one vibration.

Wavelength is the distance between any two nearest particles of the medium vibrating in the same phase.

Wavelength is the distance between two consecutive crests or troughs.

Wavelength is the distance between two consecutive compressions or rarefactions.

(*vi*) Frequency (ν). Frequency of a wave is the number of complete wavelengths travelled by the wave in one second.

(*vii*) Time Period (T). Time period of a wave is the time taken by the wave to travel a distance equal to one wavelength.

5.8. RELATION BETWEEN FREQUENCY AND TIME PERIOD

Frequency of wave, ν = Frequency of vibration of the particles of the medium

Time period of wave, T = Time period of vibration of the particles of the medium

Time taken to complete ν vibrations is 1 second.

Time taken to complete 1 vibration is $\frac{1}{\nu}$ second. But this time is equal to time period T.

$$\therefore \qquad T = \frac{1}{\nu} \qquad \text{or} \qquad \nu = \frac{1}{T} \qquad \text{or} \qquad \nu T = 1$$

5.9. RELATION BETWEEN VELOCITY, FREQUENCY AND WAVELENGTH OF A WAVE

Distance travelled by wave in time T = λ

Distance travelled by wave in unit time = $\frac{\lambda}{T}$

$$\text{or} \qquad \upsilon = \frac{\lambda}{T} \qquad \text{or} \qquad \boxed{\upsilon = \nu\lambda}$$

So, velocity of wave is the product of frequency and wavelength of the wave. This relation holds for longitudinal as well as transverse waves.

5.10. SOUND

Hearing, like sight, touch, taste etc. is a primary sensation. The term 'sound' is used in two ways. One is the sensation of hearing and another is the physical cause which produces that sensation. When we say that we hear the sound of chirping birds, we refer to this sensation. But when we say that sound travels in air at a speed of 340 m s^{-1}, we refer to the waves of sound which are external to our system of hearing. This is the physical sense in which we use the term 'sound'. The other one is the physiological sense in which we use the term 'sound'.

Sound *may be defined as the physical cause which enables us to have the sensation of hearing.*

Both sound and light are associated with wave motion. Light waves are electromagnetic waves propagating in free space at a tremendous speed of three lakh kilometre per second. On the other hand, sound is a mechanical wave motion, in an elastic medium, moving with a small speed of about 340 m s^{-1} nearly. Further, whereas light does not require any medium to pass through, sound cannot travel in vacuum.

Sound is produced by the vibrations of sounding body. Our ear is not sensitive to all such vibrations. Our range of hearing, *i.e., audible range* is from 20 Hz to 20,000 Hz. Any vibration with a frequency greater than 20,000 Hz is called an *ultrasonic vibration.* A bat produces ultrasonic vibrations which are beyond the range of human hearing. The word 'ultrasonic' should not be confused with supersonic. Any object moving with a speed greater than the speed of sound is said to move with a *supersonic speed.*

Sound requires a material medium for propagation. If there is no material medium between two points as in vacuum, sound cannot travel from one point to another.

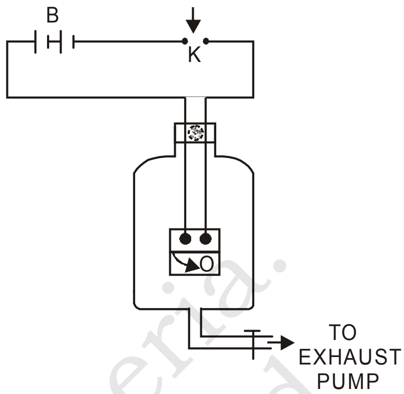


Fig. 5.8. Sound does not travel in vacuum

Example 1. *The audible frequency range of a human ear is 20 Hz – 20 kHz. Convert this into the corresponding wavelength range. Take the speed of sound in air at ordinary temperature to be 340 m s^{-1}.*

Solution. Lower limit of wavelength, $\lambda_{min.} = \dfrac{v}{\nu_{max.}}$

or $$\lambda_{min.} = \frac{340 \text{ m s}^{-1}}{20 \times 10^3 \text{ Hz}} = 17 \times 10^{-3} \text{m} = \mathbf{17 \text{ mm}}$$

Upper limit of wavelength, $\lambda_{max.} = \dfrac{v}{\nu_{min.}} = \dfrac{340 \text{ m s}^{-1}}{20 \text{ s}^{-1}} \text{m} = \mathbf{17 \text{ m}}$

Example 2. *An observer standing at a sea-coast observes 54 waves reaching the coast per minute. If the wavelength of the waves is 10 m, find the velocity of the waves.*

Solution. $$\nu = \frac{54}{60} \text{s}^{-1}; \ \lambda = 10 \text{ m}$$

$$v = \nu\lambda = \frac{54}{60} \times 10 \text{ m s}^{-1} = \mathbf{9 \text{ m s}^{-1}}$$

5.11. SPEED OF WAVE MOTION

(*i*) The speed of **transverse wave in a solid** is given by:

$$v = \sqrt{\frac{\eta}{\rho}},$$

where η is the modulus of rigidity of the material and ρ is its density.

(*ii*) The speed of **transverse waves in a stretched string** is given by:

$$v = \sqrt{\frac{T}{\mu}}$$

where T is the tension in the string and μ is the linear mass density, *i.e., mass per unit length* of the string. In SI units, T is measured in newton and 'μ' in kg m^{-1}.

Let diameter of a wire = D ; Density of material of wire = ρ.

Then, μ = mass per unit length of wire

= volume of unit length × density

= cross-sectional area × unit length × density

$$= \pi\left(\frac{D}{2}\right)^2 \times 1 \times \rho$$

$$\therefore \qquad v = \sqrt{\frac{T}{\pi\left(\frac{D^2}{4}\right)\times\rho}} = \frac{2}{D}\sqrt{\frac{T}{\pi\rho}}$$

(*iii*) Speed of longitudinal waves in solids, liquids and gases

Newton, on the basis of theoretical considerations, deduced the following formula for the velocity of longitudinal waves in an elastic medium.

$$v = \sqrt{\frac{E}{\rho}}$$

where E is the elasticity of the medium and ρ is the density of the undisturbed medium. In the case of solids, E represents the Young's modulus of elasticity. In the case of liquids and gases, E represents the bulk modulus of elasticity.

(*iv*) When sound waves propagate through **a long thin rod**, the length of the rod decreases in the region of compression and increases in the region of rarefaction. The only type of strain involved in this is '*longitudinal strain*'. Therefore, the only modulus of elasticity to be considered in this case is 'Young's modulus of elasticity'. The velocity of sound in a long thin rod is given by,

$$v = \sqrt{\frac{Y}{\rho}}$$

Here, ρ is the density of the material of the rod.

(*v*) The velocity of sound in a **liquid** is given by

$v = \sqrt{\frac{B}{\rho}}$ where B is the bulk modulus of elasticity and ρ is the density of the liquid.

Example 3. *Find the speed of transverse waves in a copper wire having a cross-sectional area of 1 mm² under the tension produced by 1 kg wt. The relative density of copper = 8.93.*

Solution.

$$a = 1\ \text{mm}^2 = 10^{-6}\ \text{m}^2,$$

$$\rho = 8.93 \times 10^3\ \text{kg m}^{-3}$$

$$T = 1\ \text{kg wt} = 9.8\ \text{N},$$

mass/length,

$$\mu = 10^{-6} \times 1 \times 8.93 \times 10^3\ \text{kg m}^{-1}$$

$$= 8.93 \times 10^{-3}\ \text{kg m}^{-1}$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{9.8}{8.93 \times 10^{-3}}}\ \text{m s}^{-1} = \mathbf{33.13\ m\ s^{-1}}$$

Example 4. *Deduce the velocity of longitudinal waves in a metal rod. Given : modulus of elasticity = 7.5 × 10¹⁰ N m⁻² and density = 2.7 × 10³ kg m⁻³.*

Solution.

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{7.5 \times 10^{10}}{2.7 \times 10^3}}\ \text{m s}^{-1}$$

$$= \mathbf{5.27 \times 10^3\ m\ s^{-1}}$$

Example 5. *Determine the speed of sound in a liquid of density 8000 kg m⁻³. Given : bulk modulus = 2 × 10⁹ N m⁻².*

Solution.

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2 \times 10^9}{8000}}\ \text{m s}^{-1} = \mathbf{500\ m\ s^{-1}}$$

5.12. NEWTON'S FORMULA FOR THE VELOCITY OF SOUND WAVES IN AIR

Newton assumed that sound waves travel in air under isothermal conditions, *i.e.*, temperature remains constant. So, the changes in pressure and volume obey Boyle's law.

$\therefore$ PV = constant

Differentiating, $PdV + VdP = 0$ or $PdV = -VdP$

or $$P = -\frac{dP}{dV/V} = \frac{\text{stress}}{\text{strain}} = \text{(isothermal) elasticity } B_i$$

Now, $$v = \sqrt{\frac{B_i}{\rho}} = \sqrt{\frac{P}{\rho}}$$

which is Newton's formula for the velocity of sound waves in air or in a gas.

Let us apply this formula to calculate the velocity of sound in air at NTP.

At NTP, density ρ of air = 1.293 kg m^{-3}

and pressure, P = 0.76 m of Hg column

$= 0.76 \times 13600 \times 9.8$ Nm^{-2}

($\because$ $P = hdg$ and $d_{Hg} = 13600$ kg m^{-3})

$$\therefore \quad v = \sqrt{\frac{0.76 \times 13600 \times 9.8}{1.293}} \text{ m s}^{-1} \approx 280 \text{ m s}^{-1}$$

This value is nearly 16% less than the experimental value of 332 m s^{-1}. This discrepancy could not be satisfactorily explained by Newton.

5.13. LAPLACE'S CORRECTION

Laplace, a French mathematician, suggested that *sound waves travel in air under adiabatic conditions and not under isothermal conditions* as suggested by Newton. He gave the following two reasons for this.

(i) When sound waves travel in air, the changes in volume and pressure take place rapidly. ***(ii)*** Air or gas is a bad conductor of heat.

Due to both these factors, the compressed air becomes warm and stays warm whereas the rarefied air suddenly cools and stays cool. For adiabatic changes in pressure and volume,

$$PV^{\gamma} = \text{constant}$$

On differentiation, $P\gamma V^{\gamma-1}dV + V^{\gamma}dP = 0$

or $$\gamma P = -\frac{V^{\gamma}dP}{V^{\gamma-1}dV} = -\frac{VdP}{dV} = -\frac{dP}{\frac{dV}{V}} = B_a,$$

where B_a is adiabatic elasticity.

Now, $$v = \sqrt{\frac{B_a}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$$

which is Laplace's corrected formula for velocity of sound waves in air or gas.

Again, $v = \sqrt{\gamma} \times \sqrt{\frac{P}{\rho}} = \sqrt{1.41} \times 280 \text{ m s}^{-1} = 332.5 \text{ m s}^{-1}$

This result agrees very well with the experimental value of 332 m s^{-1}. This establishes the correctness of Laplace's formula.

5.14. FACTORS AFFECTING THE VELOCITY OF SOUND IN GASES

(*i*) Effect of change in pressure

At constant temperature, PV = constant (Boyle's law)

or $$\frac{Pm}{\rho} = \text{constant}$$

where m is the mass of the gas and ρ is its density.

or $$\frac{P}{\rho} = \text{constant} \qquad [\because m \text{ is constant.}]$$

or $$\frac{\gamma P}{\rho} = \text{constant} \qquad [\because \gamma \text{ is also constant.}]$$

$\therefore \quad v \left(= \sqrt{\frac{\gamma P}{\rho}} \right)$ is also constant.

So, *if the temperature remains constant, the change in pressure has no effect on the velocity of sound in a gas.*

Clearly, the velocity of sound in a gas is independent of pressure, provided temperature remains constant.

(*ii*) Effect of change in temperature

Let v_0 and v_t be the velocity of sound in a gas 0°C and t°C respectively. Let γ and P remain the same at both temperatures.

Thus, $$v_0 = \sqrt{\frac{\gamma P}{\rho_0}} \quad \text{and} \quad v_t = \sqrt{\frac{\gamma P}{\rho_t}}$$

Dividing, $\dfrac{v_t}{v_0} = \sqrt{\dfrac{\gamma P}{\rho_t}} \times \sqrt{\dfrac{\rho_0}{\gamma P}} = \sqrt{\dfrac{\rho_0}{\rho_t}}$ ...(1)

Let V_0 and ρ_0 be the volume and density respectively of a given mass *m* of gas at 0°C. Let V_t and ρ_t be the volume and density respectively for the same mass *m* of gas at *t*°C.

Then, $V_t\rho_t = V_0\rho_0 = m$ or $\dfrac{V_t}{V_0} = \dfrac{\rho_0}{\rho_t}$

But $\dfrac{V_t}{V_0} = \dfrac{T}{T_0}$ (Charle's law)

where T_0 and T are the absolute temperatures corresponding to 0°C and *t*°C respectively.

$\therefore \dfrac{T}{T_0} = \dfrac{\rho_0}{\rho_t}$ $\therefore \dfrac{v_t}{v_0} = \sqrt{\dfrac{T}{T_0}}$ ...(2) [from equation (1)]

So, the velocity of sound varies directly as the square root of the absolute temperature of the gas. This explains as to why sound travels faster on a hot summer day than on a cold winter day.

Temperature coefficient of velocity of sound

From equation (2), $\dfrac{v_t}{v_0} = \sqrt{\dfrac{273+t}{273+0}} = \sqrt{\dfrac{273+t}{273}}$

or $\dfrac{v_t}{v_0} = \sqrt{1+\dfrac{t}{273}} = \left(1+\dfrac{t}{273}\right)^{1/2}$

Assume *t* to be small. Expanding the right hand side of the above equation by Binomial theorem and neglecting squares and higher powers of $\dfrac{t}{273}$, we get

$$\frac{v_t}{v_0} = 1 + \frac{1}{2} \times \frac{t}{273} = 1 + \frac{t}{546}$$

or $$v_t = v_0\left(1+\frac{t}{546}\right) = v_0 + \frac{v_0}{546}t$$

or $$v_t - v_0 = \frac{v_0 t}{546} = 332 \times \frac{t}{546}\ \text{m s}^{-1} \qquad [v_0 = 332\ \text{m s}^{-1}]$$

or $$v_t - v_0 = 0.608 \times t\,\text{m s}^{-1} = 0.61 \times t\,\text{m s}^{-1}$$

Temperature coefficient of velocity of sound,

$$\alpha = \frac{v_t - v_0}{t} = 0.61 \text{ m s}^{-1}\,{}^\circ\text{C}^{-1}$$

When $t = 1^\circ$C, then $v_t - v_0 = 0.61 \text{ m s}^{-1}$ or 61 cm s^{-1}

So, the velocity of sound increases by 0.61 m s^{-1} for every one degree centigrade rise of temperature. This is known as the temperature coefficient of velocity of sound in air.

(*iii*) Effect of change in density

Consider two different gases at the same temperature and pressure with different densities.

Then, $v_1 = \sqrt{\frac{\gamma_1 P}{\rho_1}}$ and $v_2 = \sqrt{\frac{\gamma_2 P}{\rho_2}}$ or $\frac{v_1}{v_2} = \sqrt{\frac{\gamma_1}{\gamma_2} \times \frac{\rho_2}{\rho_1}}$

For diatomic gases, $\gamma_1 = \gamma_2$. $\therefore \frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$

So, the velocity of sound in a gas is inversely proportional to the square root of the density of the gas.

Illustration. The density of oxygen is 16 times the density of hydrogen.

$$\therefore \quad \frac{v_H}{v_{O_2}} = \sqrt{\frac{\rho_{O_2}}{\rho_H}} = \sqrt{\frac{16\rho_H}{\rho_H}} = 4$$

Thus, all other things being equal, sound travels four times faster in hydrogen than in oxygen.

(*iv*) Effect of humidity

We know that humid air contains a large proportion of water vapour. So, the density ρ_m of moist air is less than the density ρ_d of dry air.

$$\frac{\rho_d}{\rho_m} = 1.6 \qquad \text{Also, } \frac{\gamma_m}{\gamma_d} = 0.9$$

Let v_m and v_d be the velocities of sound in moist air and dry air respectively.

Then, $v_m = \sqrt{\frac{\gamma_m P}{\rho_m}}$ and $v_d = \sqrt{\frac{\gamma_d P}{\rho_d}}$

$$\frac{v_m}{v_d} = \sqrt{\frac{\gamma_m P}{\rho_m}} \times \sqrt{\frac{\rho_d}{\gamma_d P}} = \sqrt{\frac{\gamma_m}{\gamma_d} \times \frac{\rho_d}{\rho_m}}$$

or $$\frac{v_m}{v_d} > 1 \quad \text{or} \quad v_m > v_d$$

So, sound travels faster in moist air than in dry air. This explains as to why sound travels faster on a rainy day than on a dry day.

(v) Effect of wind

Let wind travel with a velocity w making an angle θ with the direction of propagation of sound [Fig. 5.9]. Then, the effective velocity of sound will be ($v + w \cos \theta$).

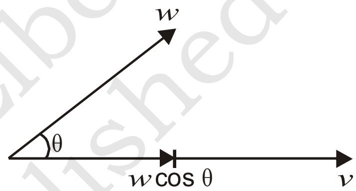


Fig. 5.9. Effect of wind on velocity of sound

If the wind blows in the direction of sound, then the velocity of sound will be increased from v to ($v + w$). If the wind blows in a direction opposite to the direction of propagation of sound, then the velocity of sound is decreased from v to ($v - w$). If wind blows perpendicular to the direction of sound, then $\theta = 90°$ and $\cos \theta = \cos 90° = 0$. So, there will be no effect on velocity of sound.

Example 6. *At what temperature will the velocity of sound in hydrogen be twice as much as that at 27°C ?*

Solution. $$\frac{v_t}{v_{27}} = \sqrt{\frac{273 + t}{273 + 27}}$$

or $$\frac{2 \times v_{27}}{v_{27}} = \sqrt{\frac{273 + t}{300}} \quad \text{or} \quad 4 = \frac{273 + t}{300}$$

or $$273 + t = 1200 \quad \text{or} \quad t = \mathbf{927°C}$$

Example 7. *At normal temperature and pressure, the speed of sound in air is 332 m s^{-1}. What will be the speed of sound in hydrogen (i) at normal temperature and pressure, (ii) at 819°C temperature and 4 atmospheric pressure ? Given : air is 16 times heavier than hydrogen.*

Solution. (*i*) Let v_a and v_h represent the speeds of sound in air and hydrogen respectively.

$$v_a = \sqrt{\frac{\gamma P}{d_a}} \text{ and } v_h = \sqrt{\frac{\gamma P}{d_h}}$$

Now, $$\frac{v_a}{v_h} = \sqrt{\frac{d_h}{d_a}} \text{ But } \frac{d_h}{d_a} = \frac{1}{16}$$

$\therefore$ $$\frac{v_a}{v_h} = \sqrt{\frac{1}{16}} = \frac{1}{4}$$

or $$v_h = 4v_a = 4 \times 332 \text{ m s}^{-1} = \mathbf{1328 \text{ m s}^{-1}}$$

(*ii*) Pressure has no effect on the velocity of sound.

$$\frac{v_{819}}{v_0} = \sqrt{\frac{273+819}{273+0}} = \sqrt{\frac{1092}{273}} = \sqrt{4} = 2$$

or $$v_{819} = 2 \times v_0 = 2 \times 1328 \text{ m s}^{-1} = \mathbf{2656 \text{ m s}^{-1}}$$

5.15. PRINCIPLE OF SUPERPOSITION OF WAVES

Statement. ***The displacement due to a number of waves acting simultaneously at a point in a medium is the sum of the displacement vectors due to each one of them acting separately.***

Since displacements are either positive or negative, therefore, the net displacement is an algebraic sum of the individual displacements.

An interesting property of a wave is that it preserves its individuality when travelling through space. Each wave behaves as if it has nothing to do with other waves. This fact is amply illustrated by the following examples.

(*i*) In an orchestra, different musical instruments are playing simultaneously. But we can detect the note produced by an individual instrument.

(*ii*) Different radio waves cross the antenna. But we can pick up any given frequency.

These examples establish the independent behaviour of a wave. *Huygen's principle of superposition* is a natural consequence of the independent behaviour of a wave.

Consider two pulses (in a string) approaching each other as shown in Fig. 5.10 (*a*). When the pulses cross each other, they combine to produce a zero resultant throughout the string as shown in Fig. 5.10 (*b*). After crossing each other, they again begin to travel independently as if nothing had happened as shown in Fig. 5.10 (*c*).

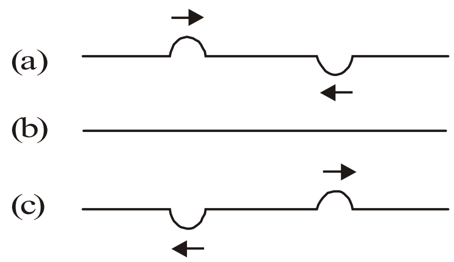


Fig. 5.10. Two pulses having equal and opposite displacements moving in opposite directions. The overlapping pulses add up to zero displacement in (*b*).

Following are the *three consequences* of the principle of superposition of waves.

(*i*) Two waves of the same frequency move with the same velocity in the same direction. This gives rise to the phenomenon of **interference of waves.**

(*ii*) Two waves of identical frequencies and amplitudes travel along the same path with the same speeds in the opposite directions. This gives rise to **stationary waves.**

(*iii*) Two waves of slightly different frequencies moving with the same velocity in the same direction give rise to the phenomenon of **beats.**

5.16. DISPLACEMENT RELATION FOR A SIMPLE HARMONIC PLANE PROGRESSIVE WAVE OR EQUATION OF PROGRESSIVE WAVE

A ***progressive wave*** *is one which travels in a given direction with constant amplitude, i.e., without attenuation.*

In the following treatment, we shall consider transverse wave motion. However, the treatment is valid for longitudinal wave motion also.

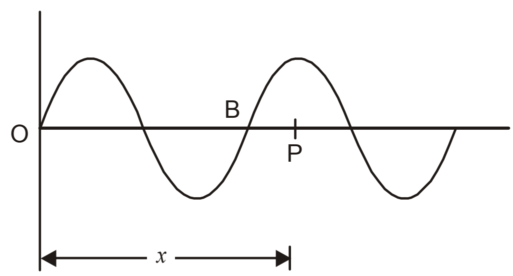


Fig. 5.11. Plane progressive wave

Let a plane wave originate at O as shown in Fig. 5.11. Let it proceed from left to right in an elastic medium. As discussed earlier, particles of the medium shall execute SHM of the same amplitude and time period about its mean position. Let us count time from the instant the particle at O crosses its mean position in the positive direction of Y-axis. The displacement y of the particle at any time t is given by

$$y(0, t) = A \sin \omega t$$

where A and ω represent the amplitude and angular frequency respectively of simple harmonic motion executed by the particle at O.

Since the disturbance is handed over from one particle to the next therefore there is a gradual fall in phase from left to right, *i.e.,* in the direction of motion. Let the phase of particle at P *lag behind* the phase of particle at O by ϕ. Then, the displacement of particle at P at any time t is given by

$$y(x, t) = A \sin (\omega t - \phi) \quad \text{...(1)}$$

At B, which is one wavelength λ apart from O, the phase difference is 2π. In other words, particles at O and B have the same phase of vibration.

At a distance λ, the phase changes by 2π.

At a distance x, the phase changes by $\frac{2\pi}{\lambda}x$.

$$\therefore \quad \phi = \frac{2\pi}{\lambda}x$$

where x is the distance of P from O.

From equation (1), $y(x, t) = A \sin\left(\omega t - \frac{2\pi}{\lambda}x\right)$ ...(2)

Now, $\omega = \frac{2\pi}{T}$ and $T = \frac{\lambda}{\upsilon}$

$$\therefore \quad \omega = \frac{2\pi}{\lambda}\upsilon$$

where υ is called the wave velocity or phase velocity.

From equation (2), $y(x, t) = A \sin\left(\frac{2\pi}{\lambda}\upsilon t - \frac{2\pi}{\lambda}x\right)$

or $$y(x, t) = A \sin \frac{2\pi}{\lambda}(\upsilon t - x) \quad \text{...(3)}$$

Also, $$y(x, t) = A \sin 2\pi\left(\frac{\upsilon}{\lambda}t - \frac{x}{\lambda}\right)$$

or $$y(x, t) = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad ...(4)$$

Again, from equation (2), $y(x, t) = A \sin(\omega t - kx)$ ...(5) $\left[\because \frac{2\pi}{\lambda} = k\right]$

Discussion. (*i*) While arriving at the wave equation, we have made a particular choice of $t = 0$. The origin of time has been chosen at an instant when the left end $x = 0$ is crossing its mean position $y = 0$ and is moving up. For a **general choice of the origin of time,** we need to add a phase constant (also known as initial phase angle) ϕ_0 so that equation (5) will be,

$$y = A \sin[(\omega t - kx) + \phi_0] \quad ...(6)$$

For $\phi_0 = \frac{\pi}{2}$, $y = A \sin\left[(\omega t - kx) + \frac{\pi}{2}\right]$

or $$y = A \cos(\omega t - kx) \quad ...(7)$$

Using $\cos(-\theta) = \cos\theta$,

$$y = A \cos(kx - \omega t) \quad ...(8)$$

For $\phi_0 = \pi$, $y = A \sin[(\omega t - kx) + \pi]$

or $y = -A \sin(\omega t - kx)$

Using $\sin(-\theta) = -\sin\theta$,

$$y = A \sin(kx - \omega t) \quad ...(9)$$

For $\phi_0 = \frac{3\pi}{2}$, $y = A \sin\left[(\omega t - kx) + \frac{3\pi}{2}\right]$

$$y = -A \cos(\omega t - kx)$$

For $\phi_0 = 2\pi$, $y = A \sin[(\omega t - kx) + 2\pi]$

or $$y = A \sin(\omega t - kx) \quad ...(10)$$

5.17. AMPLITUDE OF WAVE

The amplitude of a wave is the magnitude of maximum displacement of the constituents of the medium from their equilibrium positions as the wave passes through them.

In the equation of the travelling wave, $y(x, t)$ varies between A and – A. This is because the sine function varies between 1 and – 1. Without any loss of generality, we can take A to be a positive constant. Then A represents the maximum displacement of the constituents of the medium from their equilibrium position. Note that the displacement y may be positive or negative, but A is positive. It is called the **amplitude of the wave.**

5.18. PHASE OF WAVE

The phase of a wave is a quantity which determines the displacement of the wave at any position and at any instant. Mathematically, the quantity appearing as the argument of the sine function in the equation of the travelling wave is called the **phase of the wave**. It is denoted by ϕ.

Considering equation $y(x, t) = A \sin (\omega t - kx + \phi_0)$,

$$\phi = \omega t - kx + \phi_0$$

Clearly, ϕ_0 is the phase at $x = 0$ and $t = 0$. Hence ϕ_0 is called the initial phase angle. By suitable choice of origin on the x-axis and the initial time, it is possible to have $\phi_0 = 0$. Thus, there is no loss of generality in dropping ϕ_0 *i.e.,* in considering equations of travelling wave with $\phi_0 = 0$.

5.19. WAVELENGTH OF WAVE AND ANGULAR WAVE NUMBER

The minimum distance between two points having the same phase is called the wavelength of the wave. It is usually denoted by λ.

For simplicity, we can choose points of the same phase to be crests or troughs. The wavelength is then the distance between two consecutive crests or troughs in a wave. Considering the equation $y(x, t) = A \sin (kx - \omega t)$, the displacement at $t = 0$ is given by

$$y(x, 0) = a \sin kx$$

Since the sine function repeats its value after every 2π change in angle,

$$\therefore \qquad \sin kx = \sin (kx + 2n\pi) = \sin k\left(x + \frac{2n\pi}{k}\right)$$

That is the displacements at points x and at $x + \frac{2n\pi}{k}$ are the same, where n = 1, 2, 3, ... The least distance between points with the same displacement (at any given instant of time) is obtained by taking n = 1. λ is then given by

$$\lambda = \frac{2\pi}{k} \quad \text{or} \quad k = \frac{2\pi}{\lambda}$$

k is the angular wave number or propagation constant. Its SI unit is radian per metre or rad m^{-1}. Sometimes, k is simply measured in m^{-1}. *Angular wave number is 2π times the number of waves that can be accomodated per unit length.*

5.20. PERIOD, ANGULAR FREQUENCY AND FREQUENCY

Time period of a wave is equal to the time taken by the wave to travel a distance equal to one wavelength. It is denoted by T.

Frequency of a wave is the number of complete wavelengths traversed by the wave in one second. It is denoted by ν.

Angular frequency of a wave is 2π times the frequency of the wave.

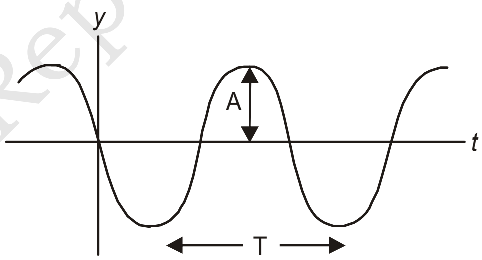


Fig. 5.12. An element of a string at a fixed location oscillates in time with amplitude A and period T, as the wave passes over it

Fig. 5.12 shows the sinusoidal plot of a travelling wave. It helps us to describe the displacement of an element (at any fixed location) of the medium as a function of time. Let us consider the equation : $y(x, t)$ = A cos $(kx - \omega t)$ and monitor the motion of the element, say at $x = 0$.

$$y(0, t) = A \sin(-\omega t)$$
$$= -A \sin \omega t$$

Now, the period of oscillation of the wave is the time it takes for an element to complete one full oscillation. That is

$$-A \sin \omega t = -A \sin \omega(t + T)$$
$$= -A \sin(\omega t + \omega T)$$

Since sine function repeats after every 2π.

$$\therefore \qquad \omega T = 2\pi \quad \text{or} \quad \omega = \frac{2\pi}{T}$$

ω is called the angular frequency of the wave. Its SI units is rad s^{-1}. The frequency ν is the number of oscillations per second. Therefore,

$$\nu = \frac{1}{T} = \frac{\omega}{2\pi}$$

ν is usually measured in hertz.

Example 8. *A wave travelling along a string is described by,*

$y(x, t) = 0.005 \sin (80.0\, x - 3.0\, t)$,

in which the numerical constants are in SI units (0.005 m, 80.0 rad m^{-1}, and 3.0 rad s^{-1}). Calculate (a) the amplitude, (b) the wavelength, and (c) the period and frequency of the wave. Also, calculate the displacement y of the wave at a distance x = 30.0 cm and time t = 20 s?

Solution. On comparing the given displacement equation with

$$y(x, t) = y_m \sin (kx - \omega t),$$

we find

(*a*) the amplitude of the wave is 0.005 m = **5 mm**

(*b*) the angular wave number k and angular frequency ω are

$$k = 80.0 \text{ rad m}^{-1} \text{ and } \omega = 3.0 \text{ rad s}^{-1}$$

We then relate the wavelength λ to k through $\lambda = 2\pi / k$

$$= \frac{2\pi \text{ rad}}{80.0 \text{ rad m}^{-1}} = \mathbf{7.85 \text{ cm}}$$

(*c*) Now we relate T to ω by the relation T = 2π/ω

$$= \frac{2\pi \text{ rad}}{3.0 \text{ rad s}^{-1}} = \mathbf{2.09 \text{ s}}$$

and frequency, ν = 1/T = **0.48 Hz**

The displacement y at x = 30.0 cm and time t = 20 s is given by

$$y = 0.005 \text{ m} \sin (80.0 \times 0.3 - 3.0 \times 20)$$

$$= 0.005 \text{ m} \sin (-36 \text{ rad}) = \mathbf{5 \text{ mm}}$$

Example 9. *Given :* $y = 0.8 \sin 16\pi \left[t + \frac{x}{40} \right]$ *metre. Calculate the wavelength and the velocity of the wave represented by this equation.*

Solution. Rewriting the given equation,

$$y = 0.8 \sin 2\pi \left[8t + \frac{8x}{40} \right] \quad \text{or} \quad y = 0.8 \sin 2\pi \left[8t + \frac{x}{5} \right]$$

Comparing with $y = A \sin 2\pi\left[\frac{t}{T} + \frac{x}{\lambda}\right]$, we get

$$\frac{1}{T} = 8 \quad \text{or} \quad \nu = 8 \text{ Hz}, \lambda = \mathbf{5\ m}$$

Velocity, $v = \nu\lambda = \mathbf{40\ m\ s^{-1}}$

5.21. FUNDAMENTAL MODE AND HARMONICS OF A STRING

(*i*) First or Fundamental mode of vibration. In this mode of vibration, the string vibrates as a whole in one segment (Fig. 5.13*a*). There are two nodes and one antinode. If λ_1 is the wavelength of the standing wave, then $\frac{\lambda_1}{2}$ = L or λ_1 = 2L. The corresponding frequency of vibration is given by

$$\nu_1 = \frac{v}{\lambda_1} = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

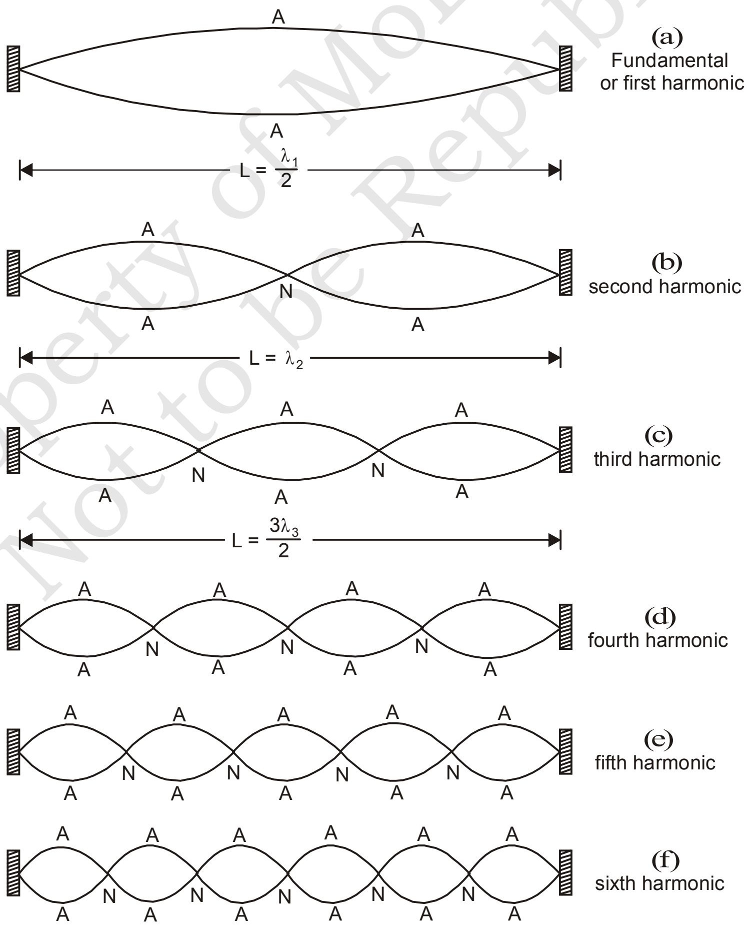


Fig. 5.13. Stationary waves in a stretched string fixed at both ends.

This is the lowest possible natural frequency of the string. This frequency is called fundamental frequency. The sound or note produced is called ***fundamental note*** or ***fundamental tone*** or ***first harmonic.***

(*ii*) Second mode of vibration. In this mode of vibration, the string vibrates in two segments or loops of equal length (Fig. 5.13*b*). There are three nodes and two antinodes. If λ_2 is the wavelength of the standing wave, then λ_2 = L. The corresponding frequency of vibration is given by

$$\nu_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2\left(\frac{v}{2L}\right)$$

$$= 2\nu_1 = 2\left[\frac{1}{2L}\sqrt{\frac{T}{\mu}}\right]$$

The frequency of vibration of the string becomes twice the fundamental frequency. The note produced is called **first overtone** or **second harmonic**.

(*iii*) Third mode of vibration. In this mode of vibration, the string vibrates in three segments or loops of equal length (Fig. 5.13*c*). If λ_3 is the wavelength, then $L = \frac{3\lambda_3}{2}$ or $\lambda_3 = \frac{2L}{3}$. The corresponding frequency is

$$\nu_3 = \frac{v}{\lambda_3} = \frac{3v}{2L} = 3\left(\frac{v}{2L}\right) = 3\nu_1 = 3\left[\frac{1}{2L}\sqrt{\frac{T}{\mu}}\right]$$

The frequency of vibration of the string becomes three times the natural frequency. The note produced is called **second overtone** or **third harmonic**.

Figs. 5.13(*d*), (*e*) and (*f*) show fourth, fifth and sixth mode of vibration.

In general, if the string is made to vibrate in n loops or segments, then $L = n\frac{\lambda_n}{2}$ or $\lambda_n = \frac{2L}{n}$. $\nu_n = \frac{v}{\lambda_n} = n\frac{v}{2L}$ or $\nu_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$

Positions of Nodes. In the first mode, there are two nodes. These are located at x = 0, L. In the second mode, there are three nodes. These are located at $x = 0, \frac{L}{2}$, L. In the third mode, there are four nodes located at $x = 0, \frac{L}{3}, \frac{2L}{3}$, L. In the nth mode, there will be $(n + 1)$ nodes located at $x = 0, \frac{L}{n}, \frac{2L}{n}, \frac{3L}{n}, \ldots\ldots, L$.

Positions of Antinodes. In the first mode, there is one antinode located at $x = \frac{L}{2}$. In the second mode, there are two antinodes located at $x = \frac{L}{4}, \frac{3L}{4}$. In the third mode, there are three antinodes located at $x = \frac{L}{6}, \frac{3L}{4}, \frac{5L}{6}$. In the *n*th mode, there are *n* antinodes located at $x = \frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n}, \ldots, \frac{(2n-1)L}{2n}$.

5.22. LAWS OF VIBRATIONS OF STRINGS

We know that $$\nu = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

The following laws of vibrations of strings follow from this equation.

(*i*) Law of length. If the tension in a given string remains constant, then the fundamental frequency varies inversely as the length.

$$\nu \propto \frac{1}{L}$$

If the length of the string is halved, the frequency is doubled.

(*ii*) Law of tension. For a string of given length and material, the fundamental frequency varies directly as the square root of the tension.

$$\nu \propto \sqrt{T}$$

If the tension is increased four times, the frequency of the note becomes double.

(*iii*) Law of mass. For a string of given length and fixed tension, the frequency varies inversely as the square root of linear density (mass per unit length) of the string.

$$\therefore \qquad \nu \propto \frac{1}{\sqrt{\mu}}$$

If linear density is quadrupled, the frequency is halved.

Consider a string of diameter D. Let ρ be the density of material of the string.

Cross-sectional area of the string = $\frac{\pi D^2}{4}$

Volume of unit length of string = $\frac{\pi D^2}{4} \times 1 = \frac{\pi D^2}{4}$

Mass per unit length = Volume of unit length × density

$$\therefore \qquad \mu = \frac{\pi D^2}{4} \times \rho$$

$$\therefore \qquad \nu = \frac{1}{2L}\sqrt{\frac{T \times 4}{\pi D^2 \rho}} = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$$

This leads to following two laws. Of course, both these laws are contained in the *law of mass* stated earlier.

1. Law of diameter. For a string of given length and tension, the frequency is inversely proportional to the diameter of the string.

$$\nu \propto \frac{1}{D}$$

So, thinner the string, higher is the frequency of vibration.

2. Law of density. For a string of given length, diameter and tension, the frequency is inversely proportional to the square root of the density of the material of the string.

$$\nu \propto \frac{1}{\sqrt{\rho}}$$

Smaller the density, higher is the frequency of vibration.

Example 10. *A steel wire 0.72 m long has a mass of 5.0 × 10^{-3} kg. If the wire is under a tension of 60 N, what is the speed of transverse waves in the wire ?*

Solution. Mass per unit length of wire,

$$\mu = \frac{5.0 \times 10^{-3}\ \text{kg}}{0.72\ \text{m}} = 6.9 \times 10^{-3}\ \text{kg m}^{-1}$$

Tension, $T = 60$ N

Speed of wave on the wire, $v = \sqrt{\frac{T}{\mu}}$

$$= \sqrt{\frac{60}{6.9 \times 10^{-3}}}\ \text{m s}^{-1} = \mathbf{93.25\ m\ s^{-1}}$$

Example 11. *A 100 cm long wire of mass 40 g supports a mass of 1.6 kg as shown in Fig. 5.14. Find the fundamental frequency of the portion of the string between the wall and the pulley. Take g = 10 m s*$^{-2}$.

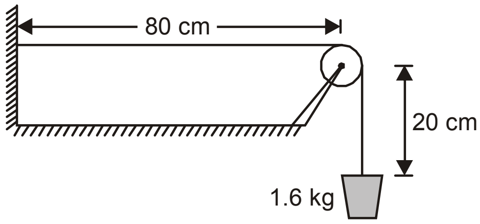


Fig. 5.14

Solution. T = 1.6 kg wt = 1.6 × 10 = 16 N

$$\mu = \frac{40 \times 10^{-3}}{1} = 0.04 \text{ kg m}^{-1}$$

$$L = (100 - 20) \text{ cm} = 0.8 \text{ m}$$

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2 \times 0.8}\sqrt{\frac{16}{0.04}} \text{ Hz} = \mathbf{12.5 \text{ Hz}}$$

Example 12. *A sonometer wire carries a brass weight (specific gravity = 8) at its end and has a fundamental frequency of 320 Hz. What would be its frequency if this weight is completely immersed in water?*

Solution. When the weight is immersed in water, buoyancy is $\frac{T}{8}$, where T is the tension in the wire.

$$\text{Net tension} = T - \frac{T}{8} = \frac{7T}{8}$$

$$\nu = 320\sqrt{\frac{7}{8}} \text{ Hz} = \mathbf{293.3 \text{ Hz}}$$

5.23. VIBRATIONS OF AIR COLUMN

Open Organ Pipe

(*i*) Introduction. It is a wind instrument in which sound is produced by setting into vibrations an air column in it.

(*ii*) Construction. It consists of a wooden or metallic hollow tube called resonator (R). A narrow tapering opening called mouth-piece (m) is provided at one end of the resonator as shown in Fig. 5.15. A slanting solid called bevel (B) is fitted near the mouth-piece. The height of the bevel is

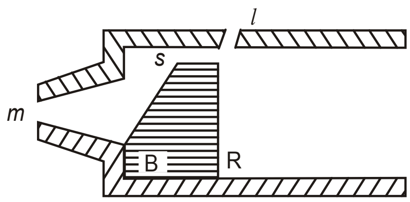


Fig. 5.15. Open organ pipe

such that there is only a narrow slit *s* between the bevel and the wall of the resonator. A sharp edge (*l*) is provided in the wall of the resonator. This is called the lip of the pipe.

(*iii*) Working. Air is blown into the pipe through the mouth-piece. After striking against the bevel, the air passes through the narrow slit *s* in the form of a thin sheet. This fast moving sheet of air strikes against the lip setting it into vibrations. The vibrating lip produces a sound called edge tone. The frequency of the edge tone depends not only on the pressure with which air is blown into the pipe but also on the distance of the lip from the slit.

(*iv*) Formation of longitudinal stationary waves. When the waves reach the open end of the pipe, they are reflected. This is because the air outside the resonator is rarer than the air inside it. The reflected and the incident waves superpose to give **longitudinal stationary waves** with fixed nodes and antinodes. When the frequency of the vibrating air column in the resonator becomes equal to the frequency of the edge tone, resonance occurs and hence loud sound is produced.

Fundamental Mode and Harmonics of Open Organ Pipe

Since both the ends of the pipe are open therefore the waves are reflected from these ends. However, the particles continue to move in the same direction even after the reflection of the waves at the open ends. So, the particles have maximum displacements at the open ends. Thus, antinodes are formed at the open ends.

Fundamental or First normal mode of vibration

This is the simplest mode of vibration in which the antinodes at the ends are separated by a node in the middle.

In this mode of vibration,

$$\frac{\lambda_1}{2} = L \qquad \text{or} \quad \lambda_1 = 2L$$

Frequency, $$\nu_1 = \frac{v}{\lambda_1} \qquad \text{or} \quad \nu_1 = \frac{v}{2L}$$

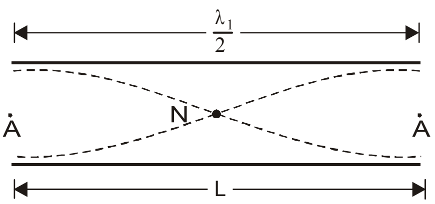


Fig. 5.16. First mode of vibration

Since this is the simplest mode of vibration therefore the sound produced is called **fundamental tone** or **first harmonic.** Longer the resonator, lesser will be the frequency of sound produced.

Second normal mode of vibration

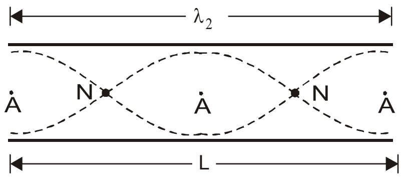


Fig. 5.17. Second mode of vibration

In this mode of vibration, the antinodes at the open ends are separated by two nodes and one antinode [Fig. 5.17].

If L be the length of the resonator, then

$$\lambda_2 = L$$

Frequency, $$\nu_2 = \frac{v}{\lambda_2}$$

or $$\nu_2 = 2 \times \frac{v}{2L} \quad \text{or} \quad \nu_2 = 2\nu_1$$

The sound produced in this mode of vibration is called **first overtone** or **second harmonic**. The frequency of first overtone is two times the frequency of the fundamental tone.

Third normal mode of vibration

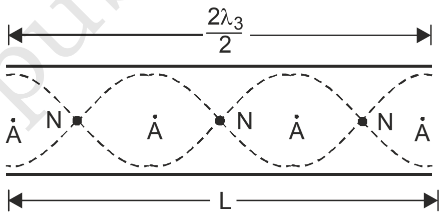


Fig. 5.18. Third mode of vibration

In this mode of vibration, the antinodes at the open ends are separated by three nodes and two antinodes.

In this mode of vibration,

$$\frac{3\lambda_3}{2} = L \quad \text{or} \quad \lambda_3 = \frac{2L}{3}$$

Frequency, $$\nu_3 = \frac{v}{\lambda_3} = \frac{v}{2L/3}$$

or $$\nu_3 = 3 \times \frac{v}{2L} \quad \text{or} \quad \nu_3 = 3\nu_1$$

The sound produced in this mode of vibration is called **second overtone** or **third harmonic**. Its frequency is three times the fundamental frequency.

By adjusting the pressure with which air is blown into the pipe, the tones of frequencies ν_1, $2\nu_1$, $3\nu_1$, $4\nu_1$, ... can be produced. Thus, the frequencies of different overtones are simple integral multiples of the frequency of fundamental tone.

In general, the frequency of vibration in *n*th normal mode of vibration in an open organ pipe is given by:

$$\nu_n = n\nu_1$$

The note produced in this case is called nth harmonic or $(n - 1)$th overtone. It would contain n nodes and $(n + 1)$ antinodes.

Closed Organ Pipe

Construction. Its construction is similar to that of open organ pipe except that its one end is closed. The waves are reflected from the closed end as the closed end behaves like a denser medium. The incident and the reflected waves superpose to form longitudinal stationary waves having fixed nodes and antinodes. When the frequency of the edge tone is equal to the frequency of vibration of the air column, then the resonance takes place. Consequently, a loud sound is heard.

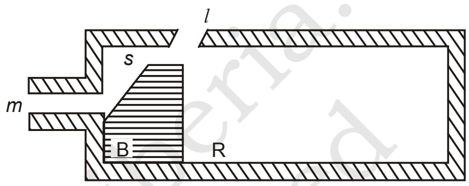


Fig. 5.19. Closed organ pipe

Fundamental Mode and Harmonics of Closed Organ Pipe

When the wave is reflected from the closed end, the direction of motion of the particles changes. So, the displacement is zero at the closed end. Thus, a node is formed at the closed end. On the other hand, an antinode is formed at the open end. This is because the displacement of particles is maximum at the open end.

Fundamental or First normal mode of vibration

This is the simplest mode of vibration in which there is a node at the closed end and an antinode at the open end [Fig. 5.20].

If L be the length of the resonator, then

$$L = \frac{\lambda_1}{4} \quad \text{or} \quad \lambda_1 = 4L$$

Frequency, $\nu_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$

Since this is the simplest mode of vibration therefore the sound produced is called **fundamental tone** or **first harmonic**. Longer the resonator, lesser will be the frequency of sound produced.

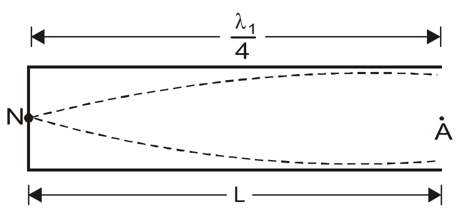


Fig. 5.20. First mode of vibration

Second normal mode of vibration

In this mode of vibration, there is one antinode and one node between a node at the closed end and an antinode at the open end [Fig. 5.21].

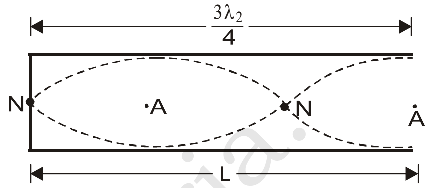


Fig. 5.21. Second mode of vibration

In this case, $\frac{3\lambda_2}{4} = L$ or $\lambda_2 = \frac{4L}{3}$

Frequency, $\nu_2 = \frac{v}{\lambda_2} = \frac{v}{4L/3}$

or $\boxed{\nu_2 = 3 \times \frac{v}{4L}}$ or $\boxed{\nu_2 = 3\nu_1}$

The sound produced in this mode of vibration is called **first overtone** or **third harmonic**. The frequency of the first overtone is three times the frequency of the fundamental tone.

Third normal mode of vibration

In this mode of vibration, there are two nodes and two antinodes between a node at the closed end and an antinode at the open end [Fig. 5.22].

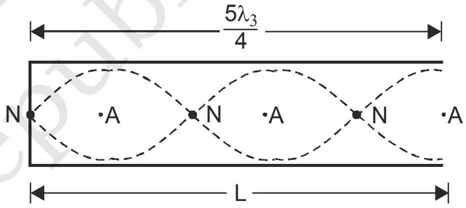


Fig. 5.22. Third mode of vibration

In this case, $L = \frac{5\lambda_3}{4}$ or $\lambda_3 = \frac{4L}{5}$

Frequency, $\nu_3 = \frac{v}{\lambda_3} = \frac{v}{4L/5}$

or $\nu_3 = 5 \times \frac{v}{4L}$ or $\nu_3 = 5\nu_1$

The sound produced in this case is called **second overtone** or **fifth harmonic**. The frequency of the second overtone is five times the frequency of the fundamental tone.

By adjusting the pressure with which air is blown into the pipe, the tones of frequencies ν_1, $3\nu_1$, $5\nu_1$,...... can be produced. Thus, the frequencies of different overtones are odd multiples of the frequency of fundamental tone.

In general, the frequency of vibration in *n*th normal mode of vibration in a closed organ pipe is given by,

$$\nu_n = (2n - 1)\frac{v}{4L} = (2n - 1)\nu_1$$

The note produced in this case is called $(2n - 1)$th harmonic or $(n - 1)$th overtone.

Comparison of closed and open organ pipes

(*i*) Fundamental note in closed pipe has half the frequency of the fundamental note in open pipe.

(*ii*) In a closed pipe, only odd harmonics are present. In an open pipe, all harmonics are present.

(*iii*) The musical sound produced by an open pipe is richer than the musical sound produced by a closed organ pipe.

Example 13. *A closed organ pipe can vibrate at a minimum frequency of 500 Hz. Find the length of the tube. Speed of sound in air = 340 m s^{-1}.*

Solution. $\nu = \frac{v}{4L}$

or $L = \frac{v}{4\nu} = \frac{340}{4 \times 500}$ m = 0.17 m = **17 cm**

Example 14. *An open organ pipe emits a note of frequency 256 Hz which is its fundamental. What would be the smallest frequency produced by a closed pipe of the same length?*

Solution. For open organ pipe, $\nu = \frac{v}{2L}$

$$256 = \frac{v}{2L} \quad \text{or} \quad v = 512\ L$$

For closed organ pipe, $\nu = \frac{v}{4L} = \frac{512\ L}{4L}$ = **128 Hz**

5.24. BEATS

When two sounding bodies of nearly the same frequency and same amplitude are sounded together, the resultant sound comprises of alternate maxima and minima.

*The phenomenon of alternate waxing and waning of sound at regular intervals is called **beats.***

The number of beats heard per second is called *beat frequency.* It is equal to the difference in the frequencies of sounding bodies. Beats are heard only when the difference in frequencies of two sounding bodies is not more than ten. This is due to *persistence of hearing.*

The time from each loud sound to the next loud sound is called ***one beat-period.***

Suppose at any place, two sound waves are in the same phase. The amplitudes of the two sound waves will be added up resulting in maximum amplitude. Since intensity is directly proportional to square of amplitude therefore loud sound will be heard.

But since the frequencies are different, even though slightly, one sound wave will start getting out of phase from the other as time passes on. Eventually, the two waves will get *out of phase* with each other. This will produce minimum amplitude resulting in a *faint sound, i.e.,* sound of low intensity. As time further elapses, the phase again goes on changing and again, we get a loud sound. In this way we continue to hear loud and faint sounds alternately. *One loud sound plus one faint sound constitute a beat.*

Analytical treatment of beats. Consider two harmonic sound waves of nearly equal frequencies ν_1 and ν_2. The periodic dips in sound, called beats, will occur with a frequency equal to $(\nu_1 - \nu_2)$.

Let a be the amplitude of each wave. Let us count time from the instant when the two sound waves are in the same phase. The displacements s_1 and s_2 at a point due to the two waves are given by

$$s_1 = a \cos 2\pi\nu_1 t \quad \text{and} \quad s_2 = a \cos 2\pi\nu_2 t$$

For the sake of simplicity, it is assumed here that there is no initial phase difference between the two wave trains. It is further assumed that the waves propagate over long distances so that the boundary effects can be neglected.

Applying the *principle of superposition of waves,*

$$s = s_1 + s_2$$

or $$s = a \cos 2\pi\nu_1 t + a \cos 2\pi\nu_2 t$$

or $$s = a(\cos 2\pi\nu_1 t + \cos 2\pi\nu_2 t)$$

or $$s = a\left(2\cos\frac{2\pi\nu_1 t + 2\pi\nu_2 t}{2}\cos\frac{2\pi\nu_1 t - 2\pi\nu_2 t}{2}\right)$$

or $$s = 2a \cos 2\pi \frac{(\nu_1 + \nu_2)}{2} t \cos 2\pi \frac{(\nu_1 - \nu_2)}{2} t$$

or $$s = \left[2a \cos 2\pi \left(\frac{\nu_1 - \nu_2}{2} \right) t \right] \cos 2\pi \left(\frac{\nu_1 + \nu_2}{2} \right) t$$

or $$s = \text{A} \cos 2\pi \left(\frac{\nu_1 + \nu_2}{2} \right) t$$

where $\text{A} \left[= 2a \cos 2\pi \left(\frac{\nu_1 - \nu_2}{2} \right) t \right]$ is the amplitude of the resultant wave. It may be noted that the frequency of the resultant wave is the average of the frequencies ν_1 and ν_2 of the superposing wave trains.

The amplitude A of the resultant wave is a function of time. A varies between + $2a$ and $-2a$. The amplitude A is *maximum, *i.e.*, + $2a$ or $-2a$ when

$$\cos \pi(\nu_1 - \nu_2)t = \pm 1$$

or $$\cos \pi(\nu_1 - \nu_2)t = \cos n\pi$$

where n = 0, 1, 2,.......

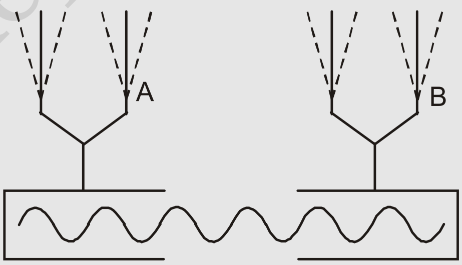
or $$\pi(\nu_1 - \nu_2)t = n\pi$$

or $$(\nu_1 - \nu_2)t = n$$

or $$t = \frac{n}{\nu_1 - \nu_2} = 0, \frac{1}{\nu_1 - \nu_2}, \frac{2}{\nu_1 - \nu_2}, \frac{3}{\nu_1 - \nu_2}, ...$$

Experimental Demonstration of Beats

Two identical tuning forks are placed on two sound boxes as shown. Attach a little wax to one prong of tuning fork A. Set the two forks into vibration. Beats will be heard. On changing the amount of wax, the number of beats per second will change.



So, the time interval between two successive maxima is $\frac{1}{\nu_1 - \nu_2}$.

Similarly, the amplitude A is minimum (zero) when

$$\cos \pi(\nu_1 - \nu_2)t = 0 \quad \text{or} \quad \cos \pi(\nu_1 - \nu_2)t = \cos(n + \frac{1}{2})\pi$$

or $$(\nu_1 - \nu_2)t = (n + \frac{1}{2})$$

*Since amplitude is maximum ∴ intensity is also maximum. It is proportional to $4a^2$.

or $$t = \frac{n + \frac{1}{2}}{\nu_1 - \nu_2} = \frac{\frac{1}{2}}{\nu_1 - \nu_2}, \frac{\frac{3}{2}}{\nu_1 - \nu_2}, \frac{\frac{2}{2}}{\nu_1 - \nu_2}, ...$$

So, the time interval between two successive minima is $\frac{1}{\nu_1 - \nu_2}$.

Thus, we find that maxima and minima occur at regular intervals of $\frac{1}{\nu_1 - \nu_2}$. So, the beat frequency is $(\nu_1 - \nu_2)$. This is equal to the difference in the frequencies of the two superposing wave trains.

Graphical representation of beats

Fig. 5.23 illustrates the phenomenon of beats for two harmonic waves of frequencies 11 Hz and 9 Hz. The amplitude of the resultant wave shows beats at a frequency of 2 Hz.

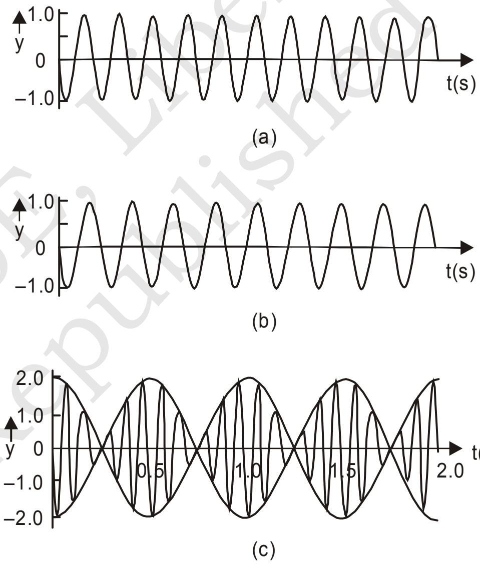


Fig. 5.23. Superposition of two harmonic waves, one of frequency 11 Hz. (*a*), and the other of frequency 9 Hz. (*b*), giving rise to beats of frequency 2 Hz, as shown in (*c*).

Uses of Beats

(*a*) To determine unknown frequency

The tuning fork of unknown frequency is sounded with a standard tuning fork of known frequency so that the beats are heard. The number of beats heard per second is determined. This is equal to the difference of 'unknown frequency' and 'known frequency'. Let N be the frequency of the standard tuning fork. Let '*a*' beats be heard per second. Then the unknown frequency is either (N + *a*) or (N – *a*).

To decide about the positive or negative sign, one of the prongs of the tuning fork of unknown frequency is **loaded with wax.** This decreases the frequency. Now, if the two tuning forks are sounded together, we will not hear '*a*' beats per second. If the number of beats heard per second is greater than *a,* then (N – *a*) was the correct frequency. If on loading, the number of beats heard per second is less than *a*, then (N + *a*) was the correct frequency of the fork.

If instead of loading one prong, it is **filed,** then the reverse results will be true.

Note that when a prong is filed a little, it becomes lighter and its frequency of vibration increases.

(*b*) Use in music. (*i*) *For tuning musical instruments.* The tension in the string of one of the two instruments is altered till beats are heard. This will occur at nearly equal frequencies. Keep on adjusting carefully till the beats disappear. Now, the two instruments are in tune. (*ii*) Sometimes in an orchestra, a deliberate 'beating' sound is produced. This gives the effect of a sonorous vibrating sound and is generally appreciated in musical performance.

(*c*) Use in electronics. Electronic beat frequency oscillators are commonly used to generate a beat frequency (BF) which is audible. Also in modern radio receivers, ultrasonic beats are generated and radio reception is obtained.

(*d*) Use in mines. The presence of dangerous gases in mines may be detected by the use of beats.

Example 15. *In an experiment, it was observed that a tuning fork and a sonometer wire gave 5 beats per second both when the length of wire was 1 m and 1.05 m. Calculate the frequency of the fork.*

Solution. Let the frequency of the fork be ν. At the smaller length of the sonometer wire (l_1 = 1 m), the frequency of the wire must be higher *i.e.,* $\nu_1 = \nu + 5$; and at the larger length (l_2 = 1.05 m), the frequency must be lower.

$$\therefore \qquad \nu_2 = \nu - 5$$

According to the law of length, $\dfrac{\nu_1}{\nu_2} = \dfrac{l_2}{l_1}$

$$\frac{\nu + 5}{\nu - 5} = \frac{1.05}{1.00}$$

On solving, we get ν = **205 Hz**

Example 16. *Two tuning forks A and B when sounded together give 4 beats/s. A is in unison with the note emitted by a 0.96 m length of a sonometer wire under a certain tension. B is in unison with 0.97 m length of the same wire under the same tension. Calculate the frequencies of the forks.*

Solution. A is in unison with a smaller length of the wire as compared to B. So, A has higher frequency as compared to B. Let ν be the frequency of A.

Then, $$\frac{\nu - 4}{\nu} = \frac{0.96}{0.97} = \frac{96}{97} \qquad \left[\because \ \nu \propto \frac{1}{l}\right]$$

or $$1 - \frac{4}{\nu} = 1 - \frac{1}{97} \quad \text{or} \quad \frac{4}{\nu} = \frac{1}{97}$$

or $$\nu = 4 \times 97 \text{ Hz} = \mathbf{388\ Hz}$$

5.25. DOPPLER EFFECT

*The apparent change in the frequency of sound when the source of sound, the observer and the medium are in relative motion is called **Doppler effect.***

Doppler effect applies to waves in general. This effect has been named after German-born Austrian Physicist Christian Johann Doppler (1803–1853).

Whenever there is relative motion between a listener (or observer) and a source of sound, the pitch or frequency of sound appears to be changed. If the source of sound is approaching the listener or the listener is approaching the source of sound or both are approaching each other, then the frequency of sound appears to be higher than the true frequency. If the source of sound is receding away from the listener or the listener is receding away from the source of sound or both are receding away from each other, then the frequency of sound appears to be lower than the true frequency.

Let us now derive expressions for the apparent frequency of sound in different cases. While deriving these expressions, we make the following **assumptions :**

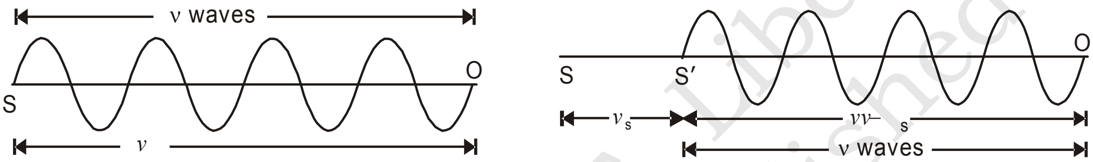
(*i*) The velocity of the source, the observer and the medium are along the line joining the positions of the source and the observer.

(*ii*) The velocity of the source and the observer is less than the velocity of sound.

(*iii*) The velocity of sound is always positive.

Case I. Source in motion, Observer at rest, Medium at rest

Suppose the source S and the observer O are separated by distance v, where v is the velocity of sound. Let ν be the frequency of the sound emitted by the source. Then, ν waves *will be emitted by the source in 1 second.* These ν waves will be accommodated in distance v [Fig. 5.24 (*a*)]. Let the source start moving towards the observer with velocity v_s. After one second, the ν waves will be crowded in distance $(v - v_s)$ [Fig. 5.24 (*b*)]. Now, the observer shall feel that he is listening to sound of wavelength λ′ and frequency ν′.



(a) Both source and observer at rest

(b) Source moving towards the observer

Fig. 5.24

Now, $$\nu' = \frac{v}{\lambda'} \quad \text{or} \quad \nu' = \frac{v}{v - v_s / \nu}$$

or $$\nu' = \frac{v\nu}{v - v_s} \quad \text{or} \quad \nu' = \frac{v}{v - v_s}\nu$$

So, as the source of sound approaches the observer, the apparent frequency ν′ becomes greater than the true frequency ν.

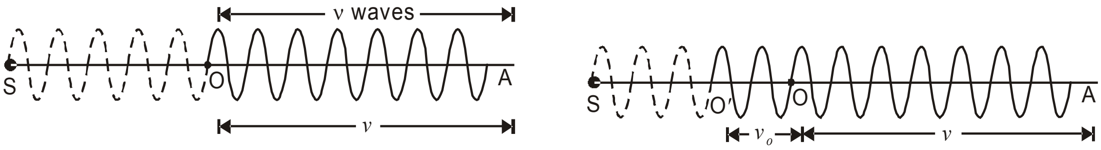
If the source is receding away from the observer, then the apparent frequency is given by

$$\nu' = \frac{v}{v + v_s}\nu$$

Case II. Observer in motion, Source at rest, Medium at rest

Let the source and observer occupy positions marked S and O respectively in Fig. 5.25 (*a*). Now take a point A such that OA = v. If both S and O are in their respective places, then ν waves given by S would be crossing O in 1 second and would fill the space OA (= v). In one second, O moves towards S with velocity v_o such that OO′ = v_o. So, the observer has received not only the ν waves occupying OA but has also received additional number of waves occupying the distance OO′. Thus in one second, the observer receives waves occupying the space AO′ such that

$$AO' = v + v_o$$



(a) Both source and observer are at rest (b) Observer moving towards source

Fig. 5.25

Number of waves in distance v = ν

Number of waves in unit distance = $\frac{\nu}{v}$

Number of waves in distance $(v + v_o)$ $= \frac{\nu}{v}(v + v_o)$

Apparent frequency, $\nu'' = \frac{\nu}{v}(v + v_o)$

$$\nu'' = \frac{v + v_o}{v}\nu$$

If the observer is moving away from the source, then the apparent frequency is given by

$$\nu'' = \frac{v - v_o}{v}\nu$$

Case III. When both the Source and Observer are moving towards each other

When the source moves towards a stationary observer,

$$\nu' = \frac{v}{v - v_s}\nu$$

Again, when the observer moves towards a stationary source,

$$\nu'' = \frac{v + v_o}{v}\nu$$

When both the source and observer move towards each other, then apparent frequency is given by

$$\nu''' = \frac{v + v_o}{v} \times \frac{v}{v - v_s}\nu \quad \text{or} \quad \nu''' = \frac{v + v_o}{v - v_s}\nu$$

If both the source and observer move in the direction of sound, then

$$\nu''' = \frac{v - v_o}{v - v_s}\nu$$

5.26. APPLICATIONS OF DOPPLER'S PRINCIPLE

(*i*) To determine the velocity of a star, galaxy etc.

Doppler's effect can be used to determine the *velocity of approach or recession of a heavenly body towards or away from the Earth.* When light from a star is examined by a spectroscope, the spectrum is found to consist of several well-defined spectral lines. If the star is approaching the Earth, a shift of spectral lines occurs towards the violet end of the spectrum. This indicates a decrease in wavelength.

When the star is receding away from the Earth, the spectral lines shift towards the red end of the spectrum indicating an increase in wavelength. These changes of wavelength on account of motion of star are called ***spectral shifts***. These help us to calculate the velocity of approach or the velocity of recession of the star.

Let the star be receding away from the Earth with velocity v. Then applying Doppler's effect, the apparent frequency of the light waves coming from the star is given by

$$\nu' = \frac{c}{c+v}\nu$$

where c is the velocity of light and ν is the true frequency of light waves.

$$\therefore \qquad \frac{\nu'}{\nu} = \frac{c}{c+v}$$

But $$\nu' = \frac{c}{\lambda'} \quad \text{and} \quad \nu = \frac{c}{\lambda}$$

where λ' and λ are the apparent wavelength and true wavelength respectively.

$$\therefore \qquad \frac{c}{\lambda'} \times \frac{\lambda}{c} = \frac{c}{c+v} \quad \text{or} \quad \frac{\lambda}{\lambda'} = \frac{c}{c+v}$$

or $$\frac{\lambda'}{\lambda} = \frac{c+v}{c} = 1 + \frac{v}{c}$$

or $$\frac{\lambda'}{\lambda} - 1 = \frac{v}{c} \quad \text{or} \quad \frac{\lambda' - \lambda}{\lambda} = \frac{v}{c}$$

or $$\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \quad \text{or} \quad \Delta\lambda = \frac{v}{c}\lambda$$

By knowing the value of $\Delta\lambda$, we can calculate the velocity v of the star with respect to Earth. It has been generally observed that the wavelength of light received from the stars shifts slightly towards the red end of the spectrum. This '**red shift**' shows that the stars are receding away from us. So, our universe is expanding.

(*ii*) Radar

It measures not only the distance and location of an aeroplane but also its velocity by determining the frequency shift.

We know that $\nu' = \dfrac{c}{c - v_s}\nu = \nu\left(\dfrac{c - v_s}{c}\right)^{-1} = \nu\left(1 + \dfrac{v_s}{c}\right) = \nu + \nu\dfrac{v_s}{c}$

or $\quad \nu' - \nu = \dfrac{v_s}{c}\nu \quad$ or $\quad \Delta\nu = \dfrac{v_s}{c}\nu \quad$ or $\quad v_s = c\dfrac{\Delta\nu}{\nu}$

So, by determining the frequency shift $\Delta\nu$, v_s can be calculated. This has to be halved to get the approach velocity of the aeroplane.

Example 17. *Determine the velocity of sound when the frequency appears to be double the actual frequency to a stationary observer.*

Solution. $\nu' = \dfrac{v}{v - v_s}\nu$

Now, $\quad \nu' = 2\nu \quad \therefore \quad 2\nu = \dfrac{v}{v - v_s}\nu$

or $\quad 2v - 2v_s = v$ or $v = 2v_s$ or $v_s = \dfrac{v}{2}$

The source should approach the stationary observer with a velocity equal to **half the velocity of sound**.

Example 18. *A factory siren whistles a note of frequency 680 Hz. A man travelling in a car at 108 km h^{-1} moving towards the factory hears the whistle. What is the apparent frequency of the sound as heard by him? Given : speed of sound in air = 340 m s^{-1}.*

Solution. $v_o = 108 \text{ km h}^{-1} = 108 \times \dfrac{5}{18} \text{ m s}^{-1} = 30 \text{ m s}^{-1}$

$v = 340 \text{ m s}^{-1}, \nu = 680 \text{ Hz}$

$\nu' = \dfrac{v + v_o}{v}\nu = \dfrac{340 + 30}{340} \times 680 \text{ Hz} = \mathbf{740\ Hz}$

Example 19. *Two railway trains, each moving with a velocity of 108 km h^{-1}, cross each other. One of the trains gives a whistle whose* *frequency is 750 Hz. What will be the apparent frequency for passengers sitting in the other train before crossing ? Given : speed of sound = 330 m s*$^{-1}$.

Solution. $v_s = 108 \text{ km h}^{-1} = 108 \times \frac{5}{18} \text{ m s}^{-1} = 30 \text{ m s}^{-1}$

$$v_o = 108 \text{ km h}^{-1} = 30 \text{ m s}^{-1}$$

Note that the source and the observer are approaching.

$\therefore$ Apparent frequency, $\nu' = \frac{v + v_o}{v - v_s}\nu$

$\therefore$ $$\nu' = \frac{330+30}{330-30} \times 750 \text{ Hz} = \mathbf{900 \text{ Hz}}$$

5.27 RELATION BETWEEN LOUDNESS AND INTENSITY

Intensity of sound represents the sound energy that flows per second across a unit area held normal to the direction of flow.

This is an objective physical definition. The feeling in the listener's mind is spoken of as loudness. Thus, a sound of high intensity possesses a greater loudness.

(*i*) According to **Weber-Fechner law**, the loudness L of sound is directly proportional to the logarithm of intensity I.

$\therefore$ $$L \propto \log I \quad \text{or} \quad L = K \log I$$

Here, K is a constant of proportionality.

(*ii*) Consider two sounds of same frequency having intensities I_1 and I_0 respectively. Let L_1 and L_0 be their corresponding loudness.

Then, $$L_1 = K \log_{10} I_1 \text{ and } L_0 = K \log_{10} I_0$$

Intensity level, $$L = L_1 - L_0 = K [\log_{10} I_1 - \log_{10} I_0]$$

or $$L = K \log_{10} \left[\frac{I_1}{I_0}\right]$$

(*iii*) Let I_0 represents the standard reference intensity (also called zero level of intensity). Its value is 10^{-12} W m^{-2}. It corresponds to the threshold audibility of a healthy human ear at a frequency of 1000 Hz.

If K = 1, then L is measured in bel. [The unit is named in honour of Alexander Graham Bell, the inventor of Telephone.]

Now, $L = \log_{10}\left[\frac{I_1}{I_0}\right]$ bel

If $I_1 = 10I_0$, then $L = \log_{10}\frac{10I_0}{I_0} = \log_{10}10 = 1$ bel

The intensity level of sound is said to be one bel if the intensity of sound is ten times the zero level of intensity.

The intensity level of sound will be 2 bel if the intensity of sound is 100 times the zero level of intensity.

(*iv*) Since bel is a large unit, therefore, a smaller unit called decibel (dB) is used.

$$1\text{dB} = \frac{1}{10}\text{ bel.}$$

Again, $L = 10\log_{10}\left[\frac{I_1}{I_0}\right]$ decibel

If $L = 1$ decibel, then $\log_{10}\left[\frac{I_1}{I_0}\right] = \frac{1}{10} = 0.1$

or $\frac{I_1}{I_0} = \text{antilog}(0.1) = 1.2589 \approx 1.26$

We can conclude from here that a 26 percent increase in the intensity raises the intensity level by 1 decibel. It is interesting to note that it is the smallest change in intensity level that a healthy human ear can detect.

If $I_1 = 100\,I_0$,

then $L = 10\log_{10}\left[\frac{100\,I_0}{I_0}\right] = 10\log_{10}100$

$= 10\log_{10}10^2 = 20\log_{10}10$

$= 20$ decibels

So, if the louder of the two sounds is 100 times more intense, then the two sounds differ by 20 decibels. Similarly, if the louder of the two sounds is 1000 times more intense, then the two sounds will differ by 30 decibels.

REVIEW EXERCISES

Do the review exercises in your notebook.

A. Multiple Choice Questions

1. A tuning fork of frequency 512 Hz makes 4 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was

(*a*) 510 Hz (*b*) 514 Hz
(*c*) 516 Hz (*d*) 508 Hz.

2. A transverse wave is represented by $y = A \sin(\omega t - kx)$.
For what value of the wavelength is the wave velocity equal to the maximum particle velocity?

(*a*) $\frac{\pi A}{2}$ (*b*) πA
(*c*) $2\pi A$ (*d*) A.

3. Two strings A and B are slightly out-tune and produce beats of frequency 5 Hz. Increasing the tension in B reduces the beat frequency to 3 Hz. If the frequency of string A is 450 Hz, calculate the frequency of string B.

(*a*) 460 Hz (*b*) 455 Hz
(*c*) 445 Hz (*d*) 440 Hz.

4. A resonance pipe is open at both ends and 30 cm of its length is in resonance with an external frequency 1.1 kHz. If the speed of sound is 330 m s^{-1} which harmonic is in resonance ?

(*a*) first (*b*) second
(*c*) third (*d*) fourth.

5. When two progressive waves $y_1 = 4 \sin(2x - 6t)$ and $y_2 = 3 \sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is

(*a*) 2 (*b*) 3
(*c*) 4 (*d*) 5.

6. A wave motion is described by $y(x, t) = a \sin(kx - \omega t)$. Then the ratio of the maximum particle velocity to the wave velocity is

(*a*) ωa (*b*) $\frac{1}{ka}$

(*c*) $\frac{\omega}{k}$ (*d*) ka.

7. Velocity of sound in air is 320 m s^{-1}. A pipe closed at one end has a length of 1 m. Neglecting end correction, the air column in the pipe cannot resonate with sound of frequency

(*a*) 80 Hz (*b*) 240 Hz

(*c*) 320 Hz (*d*) 400 Hz

8. A whistle is blown from the tower of a factory with a frequency of 220 Hz. The apparent frequency of sound heard by a worker moving towards the factory with a velocity of 30 m s^{-1} is (Velocity of sound = 330 m s^{-1})

(*a*) 280 Hz (*b*) 200 Hz

(*c*) 300 Hz (*d*) 240 Hz

9. The frequencies of two tuning forks A and B are respectively 1.5% more and 2.5% less than that of the tuning fork C. When A and B are sounded together, 12 beats are produced in 1 second. The frequency of the tuning fork C is

(*a*) 200 Hz (*b*) 240 Hz

(*c*) 360 Hz (*d*) 300 Hz

10. Two pipes are each 50 cm in length. One of them is closed at one end while the other is open at both ends. The speed of sound in air is 340 m s^{-1}. The frequency at which both the pipes can resonate is

(*a*) 680 Hz (*b*) 510 Hz

(*c*) 85 Hz (*d*) none of the above.

B. Fill in the Blanks

1. A train moving towards a hill at a speed of 72 km h^{-1} sounds a whistle of frequency 500 Hz. A wind is blowing from the hill at a speed of 36 km h^{-1}. If the speed of sound in air is 340 m s^{-1}, the frequency heard by a man on the hill is__________ .

2. When two sound sources of the same amplitude but of slightly different frequencies n_1 and n_2 are sounded simultaneously, the sound one hears has a frequency equal to __________ .

3. A travelling wave represented by $y = A \sin(\omega t - kx)$ is superimposed on another wave represented by $y = A \sin(\omega t + kx)$. The resultant is __________ .

4. Two identical piano wires, kept under the same tension T have a fundamental frequency of 600 Hz. The fractional increase in the tension of one of the wires which will lead to occurrence of 6 beats/s when both the wires oscillate together would be __________.
5. Sound waves travel at 350 m s^{-1} through warm air and at 3500 m s^{-1} through brass. The wavelength of a 700 Hz acoustic wave as it enters brass from warm air __________ .
6. Tube A has both ends open while tube B has one end closed. Otherwise they are identical. Their fundamental frequencies are in the ratio __________ .
7. The speed of sound in a gas of density ρ at a pressure P is proportional to __________ .
8. The intensity ratio of two waves at a point is $\frac{4}{9}$. The amplitude ratio will be __________ .
9. Two sound waves travel in the same direction in a medium. The amplitude of each wave is A and the phase difference between the two waves is 120°. The resultant amplitude will be __________ .
10. A plane progressive wave is given by

$$y = 2 \cos 6.284\ (330t - x).$$

The period of the wave is __________ .

C. Very Short Answer Questions

1. What is the range of frequency of audible sound?
2. Why does sound travel faster in iron than in air?
3. What kind of waves help the bats to find their way in the dark?
4. The velocity of sound in air is 332 m s^{-1}. Find the frequency of the fundamental note of an open pipe 50 cm long.
5. In which gas, hydrogen or oxygen, will sound have greater velocity?
6. In a resonance tube, the second resonance does not occur exactly at three times the length at first resonance. Why?
7. The frequency of the fundamental note of a tube closed at one end is 200 Hz. What will be the frequency of the fundamental note of a similar tube of the same length but open at both ends?
8. A wave transmits energy. Can it transmit momentum?
9. A string has a linear density of 0.25 kg m^{-1} and is stretched with a tension of 25 N. What is the velocity of the wave?
10. By how much the wave velocity increases for 1°C rise of temperature?

D. Short Answer Questions

1. A tuning fork of unknown frequency gives 4 beats with a tuning fork of frequency 310 Hz. It gives the same number of beats on filing. Find the unknown frequency.
2. The string of a violin emits a note of 540 Hz at its correct tension. The string is bit taut and produces 4 beats per second with a tuning fork of frequency 540 Hz. Find the frequency of the note emitted by this taut string.
3. The air column in a pipe closed at one end is made to vibrate in its second overtone by a tuning fork of frequency 440 Hz. The speed of sound in air is 330 ms^{-1}. Find the length of the air column. [End correction may be neglected]
4. In the following series of resonant frequencies, one frequency (lower than 400 Hz) is missing : 150, 225, 300, 375 Hz (a) What is the missing frequency? (b) What is the frequency of the seventh harmonic?
5. Flash and thunder are produced simultaneously. But thunder is heard a few second after the flash is seen. Why?

E. Long Answer Questions

1. Densities of oxygen and nitrogen are in the ratio 16 : 14. At what temperature the speed of sound in oxygen will be the same as at 15°C in nitrogen?
2. Calculate the speed of sound in oxygen from the following data. The mass of 22.4 litre of oxygen at STP (T = 273 K and P = 1.0×10^5 N m^{-2}) is 32 g, the molar heat capacity of oxygen at constant volume is $C_v = 2.5$ R and that at constant pressure is $C_p = 3.5$ R.
3. A sound wave of frequency 400 Hz is travelling in air at a speed of 320 m s^{-1}. Calculate the difference in phase between two points on the wave 0.2 m apart in the direction of travel.
4. A displacement wave is represented by

$$y = 0.25 \times 10^{-3} \sin (500t - 0.025\,x),$$

where *y, t* and *x* are in cm, second and metre respectively. Deduce (*i*) the amplitude (*ii*) the period (*iii*) the angular frequency (*iv*) the wavelength. Deduce also the amplitude of particle velocity and particle acceleration.
5. Two harmonic waves have the same displacement amplitude of 4×10^{-5} cm and their angular frequencies are 500 rad s^{-1} and 5000 rad s^{-1}. Calculate (*i*) particle velocity amplitude, and (*ii*) particle acceleration amplitude.